GABARITO

(1)

$$\begin{array}{ccc} A & x & B \\ \begin{vmatrix} 2 & -1 & 0 \\ 0 & 3 & 2 \\ 5 & 0 & 3 \end{vmatrix} & \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = & \begin{vmatrix} 2 \\ 16 \\ 21 \end{vmatrix} \end{array}$$

$Ax = B$

$x = A^{-1} \cdot B$

$$A_{11} = \begin{vmatrix} 3 & 2 \\ 0 & 3 \end{vmatrix} \qquad A^{-1} = \frac{1}{\det A} A^{t}_{cof}$$

$$A_{cof} = \begin{vmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{vmatrix} \qquad \det A = 8 \neq 0$$

$$A^{-1} = \frac{1}{8} \cdot \begin{vmatrix} 9 & 3 & -2 \\ 10 & 6 & -4 \\ -15 & -5 & 6 \end{vmatrix} = \begin{vmatrix} 9/8 & 3/8 & -2/8 \\ 10/8 & 6/8 & -4/8 \\ -15/8 & -5/8 & 6/8 \end{vmatrix}$$

$$A^{-1} \cdot B = \begin{vmatrix} 9/8 & 3/8 & -2/8 \\ 10/8 & 6/8 & -4/8 \\ -15/8 & -5/8 & 6/8 \end{vmatrix} \cdot \begin{vmatrix} 2 \\ 16 \\ 21 \end{vmatrix} = \begin{vmatrix} \frac{18+48-42}{8} \\ \frac{20+96-84}{8} \\ \frac{-30-80+126}{8} \end{vmatrix} = \begin{vmatrix} 3 \\ 4 \\ 2 \end{vmatrix}$$

(2) a) $\lim\limits_{x \to \pi/4} \dfrac{\operatorname{sen} x - \cos x}{1 - \operatorname{tg} x}$

$$\lim_{x \to \pi/4} \frac{\cos x \left(\frac{\operatorname{sen} x}{\cos x} - 1 \right)}{1 - \operatorname{tg} x} = \lim_{x \to \pi/4} \frac{\cos x (\operatorname{tg} x - 1)}{1 - \operatorname{tg} x} =$$

$$\lim_{x \to \pi/4} \frac{-\cos x \cancel{(1 - \operatorname{tg} x)}}{\cancel{(1 - \operatorname{tg} x)}} = \lim_{x \to \pi/4} -\cos x = -\cos\left(\frac{\pi}{4}\right)$$

$$\boxed{= -\frac{\sqrt{2}}{2}}$$

(2) b) $\lim_{x \to +\infty} \frac{2x^3}{x^2+1}$

$\lim_{x\to+\infty} \frac{2x \cdot \cancel{x^2}}{\cancel{x^2}\left(1+\frac{1}{x^2}\right)} = \lim_{x\to+\infty} \frac{2x}{1+\frac{1}{x^2}} =$

$\lim_{x\to+\infty} x \cdot \left(\frac{2}{1+\frac{1}{x^2}}\right) = +\infty \cdot 2 = \boxed{+\infty}$

$\lim_{x\to+\infty} x = +\infty$

$\lim_{x\to+\infty} \frac{2}{1+\frac{1}{x^2}} = 2$

(3) a) $\int \sqrt{2px}\, dx = \int \sqrt{2p} \cdot \sqrt{x}\, dx =$

$\sqrt{2p} \int \sqrt{x}\, dx = \sqrt{2p} \int x^{1/2} dx = \sqrt{2p}\, \frac{x^{1/2+1}}{\frac{1}{2}+1} + C =$

$\sqrt{2p}\, \frac{2}{3} x^{3/2} + C = \boxed{\sqrt{2p} \cdot \frac{2}{3}\sqrt{x^3} + C}$

b) $\int \frac{45}{x \cdot \ln x} dx = 45 \int \frac{1}{x \ln x} dx$

$\left.\begin{array}{l} u = \ln x \\ du = \frac{1}{x} dx \end{array}\right\}$ $\therefore 45 \int \frac{1}{u} du = 45 \ln|u| + C$

$u = \ln x$

$\int \frac{45}{x \cdot \ln x} dx = \boxed{45 \ln|\ln x| + C}$

4 a) $y = \dfrac{3x^2 - 4}{2x+5}$

$$y' = \frac{6x(2x+5) - (3x^2-4) \cdot 2}{(2x+5)^2} = \frac{12x^2 + 3x - 6x^2 + 8}{(2x+5)^2} =$$

$$\frac{6x^2 + 30x + 8}{(2x+5)^2} = \boxed{\frac{2 \cdot (3x^2 + 15x + 4)}{(2x+5)^2}}$$

b) $y = \ln(\operatorname{sen}(3x))$

$$y' = \ln'(\operatorname{sen}(3x)) \cdot \operatorname{sen}'(3x) \cdot (3x)'$$

$$= \frac{1}{\operatorname{sen}(3x)} \cdot \cos(3x) \cdot 3 = \frac{3 \cos(3x)}{\operatorname{sen}(3x)}$$

$$\boxed{y' = 3 \cdot \operatorname{cotg}(3x)}$$

5 $y = ax + b$ $V_1 = (293,432 ; 0)$
$V_2 = (859,657 ; 671,198)$

$$\begin{cases} 0 = a \cdot 293,432 + b \\ 671,198 = a \cdot 859,657 + b \end{cases}$$

$671,198 = 859,657a - 293,432a$
$671,198 = 566,225 \cdot a$
$a = 1,185$
$b = -293,432 \cdot 1,185$
$b = -347,717$ $y = 1,185x - 347,717$

$y = ax + b$ $V_3 = (1277,570 ; 807,240)$
$V_4 = (891,575 ; 1.394,602)$

$$\begin{cases} 807,240 = a \cdot 1277,570 + b \\ 1.394,602 = a \cdot 891,575 + b \quad (-1) \end{cases}$$

$-587,362 = 385,995 \cdot a$
$a = -1,522$
$1394,602 = -1522 \cdot 891,575 + b$
$b = 2751,579$ $y = -1,522x + 2751,579$

$1,185x - 347,717 = -1522x + 2751,579$

$$\boxed{\begin{array}{l} x = 1.144,825 \\ y = 1009,235 \end{array}}$$