Investigating the expectation hypothesis and the risk premium dynamics: new evidence for Brazil

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Abstract

We re-examine the validity of the Expectation Hypothesis (EH) of the term structure for the Brazilian fixed income market, using data from Jan-2000 to Jun-2017. Furthermore, we investigated the out-of-sample predictability of bond excess returns by means of common factors extracted from a cross-section of Brazilian macro-variables and zero-coupon interest rates. The EH is rejected throughout the term structure examined on the basis of the statistical tests across the entire maturity spectrum considered. Our results confirm previous findings, mostly obtained for developed markets, that a linear combination of forward rates and macroeconomic factors can explain a substantial portion of movements in bonds excess returns, contributing novel and up- to-date evidence from a large and dynamic emerging bond market, such as Brazil. Furthermore, we find that the factor extracted from a large panel of macroeconomic variables generates significant gains in forecasting bond excess returns relative to yield curve information.

Keywords: Expectation hypothesis, Bond risk premia, Factor models, Excess return predictability, Out-of-sample forecasts

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1. Introduction

The expectations hypothesis (EH) of the term structure of interest rates, the proposition that the long-term yield is determined by the market's expectation of the short-term yields over the holding period of the long-term asset plus a constant risk premium, has attracted considerable attention, both within academic and practitioner circles. The expectations hypothesis, which asserts that expected excess returns are time invariant, plays an important role in economics and finance, especially in monetary policy analyses. If the expectations theory prevails, then central banks can influence long-rates by operating at the short-end of the market. Hence, it is not surprising that the EH has been tested extensively using a wide variety of interest rates and over a variety of time periods, mostly for developed countries. The empirical studies showed that the expectations hypothesis (EH) of the term structure of interest rates is rejected by the data in the majority of cases and argues, almost unequivocally, that deviations from the EH reflect time-varying risk premia.

In this paper, we analyze the Brazilian yield curve in order to examine the validity of the expectations hypothesis for the Brazilian yield curve. In addition, we investigate the predictability of the excess returns on Brazilian zero-coupon bonds with maturities ranging from 2 to 5 years. The importance of studying Brazil is due to: i) Brazil is large and dynamic bond market, amongst the largest and most liquid emerging bond markets; ii) Brazil operates an inflation target system since 1999, so it is crucial to know whether futures interest rates react to changes in the SELIC rate (monetary policy instrument); iii) the link between inflation (and other macro-factors) and interest rate. Therefore, Brazil is one of the emerging markets that can constitute an important case study for this type of research as it has one of the largest bond markets in the world among the developing countries.

It is well known that the expectation hypothesis is rejected in favor of bond returns being predictable by forward or yield spreads. As predictors we take forward spreads of Fama & Bliss (1987), the forward factor proposed by Cochrane & Piazzesi (2005), and the macro factor proposed by Ludvigson & Ng (2009) and refer to them as FB, CP, and LN. Although less extensive than the equity return forecastability literature, various studies aim to predict government bond excess returns as well. The issue of forecasting bond returns is of great interest to academics and practitioners. For academic researchers, the interest in forecast bond returns lies mainly in understanding why investors' required risk compensation should vary over time. For investors, predictability of returns is naturally attractive from an asset management perspective. At the macroeconomic level, moreover, the EH is relevant to understand the impact of monetary policy and its transmission mechanism (Bernanke & Blinder, 1992).

The theoretical basis for controlling other interest rates through the monetary policy instrument (the current short rate) is the expectations hypothesis of the term structure of interest rates. The current short rate and future short rate expectations are closely connected to monetary policy. Risk compensation (excess return) is frequently call the “term premia”, which is the difference between the actual long yield and the Expectation Hypothesis consistent long yield. However, the EH has been rejected using a variety of interest rates, time periods, monetary policy regimes, etc. (e.g., Klein, 1990; Campbell & Shiller, 1991; Cochrane & Piazzesi, 2005; Thornton, 2005; Ludvigson & Ng, 2009; Sarno *et al.* , 2016, and the literature therein).

A number of studies indicate the presence of predictable variation in government bond excess returns. Most of these empirical studies have employed information from the term structure of spot and forward rates in order to predict bond returns.This literature finds evidence of a time-varying risk premia in bond returns. Fama & Bliss (1987) and Campbell & Shiller (1991) find that the spreads between forward and spot rates have predictive power for excess returns and its forecasting power increases with the forecast horizon. Cochrane & Piazzesi (2005) run predictive regressions of one year excess log returns by considering a combination of forward rates as predictors and find that information contained in the entire term structure of interest rates can capture up to 40% per cent of the variation of one year excess bond returns over the period from January 1964 to December 2003. Using US bonds data, Thornton & Valente (2012) evaluate the out-of-sample forecasting ability of the predictors in FB and CP in a dynamic asset allocation strategy and find that predictive models based on forward rates are unable to generate systematic economic value over the expectations hypothesis (EH) no-predictability benchmark. Sarno *et al.* (2016) find under affine term structure model framework that the evident statistical predictability of bond risk premia rarely turns into investors’ economic gain. However, Gargano *et al.* (2017), by using non-overlapping excess bond returns and models that allow for time-varying parameters and stochastic volatility in the predictive regressions, finds that statistically significant gains in out-of-sample forecasting accuracy can translate into economic value for a real-time investor.

More recent developments in this literature link the predictable component to factors whose variations lie outside the span of current yields, such as macroeconomic variables. For example, Ludvigson & Ng (2009) and Cooper & Priestley (2009) document that macro factors predict bond returns, adding incremental forecasting power in excess of information contained in yields. Moving away from yield curve information, Wright (2011) considers survey forecasts on macroeconomic fundamentals to improve term premia estimates. Joslin *et al.* (2014) provide evidence that macroeconomic variables contain rich information on yields. Eriksen (2017), using survey forecasts from Survey of Professional Forecasters, extracts proxy for expected business condition and find it consistently affects bond excess returns beyond the current term structure and macroeconomic variables. In international markets, several studies (e.g., Dahlquist & Hasseltoft, 2013; Zhu, 2015, and references therein) find that forward rates strongly predict international excess bond returns.

While most of the empirical studies focuses on the developed countries, particularly on U.S.

data, this very important literature has remained scarce for the emerging market cases. Brazil is one of the emerging market economies that can constitute an important case study for this type of research as it has one of the biggest bond markets in the world among the developing countries. Therefore, in this paper we aim to fill this gap by presenting a new research for Brazilian market fixed income. Motivated by the enormous growth of the Brazilian fixed income market over the past 15 years, we employ forward spreads, macro factors, or the term structure of forward rates as predictors to evaluate the validity of the expectations hypothesis and test the predictability of bond excess returns in Brazil.

We empirically examine the Brazilian term structure dynamics using monthly observations from January 2000 to June 2017. Our findings indeed suggest that Brazilian yield curve its not consistent with the expectations hypothesis for the data period considered in the study. We find evidence of time-varying risk-premium to all maturities. Our results suggest that macro factors do contribute substantially to the understanding of the dynamics of risk premia in the Brazilian fixed income market. The out-of-sample forecasting analysis shows that the macro, LN-factor consistently delivers significant out-of-sample gains relative to the expectations hypothesis of interest rates (the historical average). A two-factor model comprising the combination of forward rates (CP-factor) and the macro factor (LN) generates notable gains in out-of-sample forecast accuracy compared with a model based on the expectations hypothesis. The CP + FB model, witch takes into account the macro factor, provides better forecasting performance than the model with only yield curve information across the entire maturity spectrum (2-, 3-, 4-, and 5-year). For example, for long maturities, the improvement in the root mean square error (RMSE) form using unspanned macro information in forecasting excess return is about 3% on average, witch corresponds to an out-of-sample R^2 of 4.8%. This result indicates that the macro factor extracted from a large panel of macroeconomic variables contains rich information on future excess bond returns.

The forecasts turn out that the expectations hypothesis fails in the Brazilian fixed income market. Hence, the usefulness of the EH for financial market analysts and policymakers is doubtful. The existing related literature for Brazilian market is very limited. One related study by Tabak (2009) tested the expectations hypothesis (EH) using cointegration techniques, for maturities ranging from 1 to 12 months, covering the period from 1995 to 2006. They found evidence suggesting that support the EH and that the risk premium may be time-varying. Lima & Issler (2003) and Tabak & Andrade (2003), have found evidence of time-varying risk premium for the term structure of interest rates for Brazil. Lima & Issler (2007) tested the expectations hypothesis for Brazil using cointegration and found evidence contrary to EH. It is important to notice that this paper adds to the financial literature by testing the EH for Brasil, which is an emerging market economy that has one of the biggest bond market in the world among the emerging countries. Furthermore, to the best of our knowledge, this article is the first to applies the considered models to evaluate the EH and to predict excess bond returns in Brazilian fixed income market.

The outline of the paper is as follows. Section 2 introduces the EH and the models based on forward rates or forward spreads within which the empirical work is carried out. Section 3 briefly describes the data and preliminary statistics on our dataset and reports the main empirical results. Section 4 concludes the article.

2. Bond Returns, Risk Premia and The Expectations Hypothesis

The bond risk premium measures the compensation required by risk averse investors to hold long-term government bonds for facing capital loss risk, if the bond is sold before maturity.

2.1. Bond returns and forward rates

Consider an τ-period zero coupon bond paying \$1 at maturity, whose nominal price at time t is $P_t^{(\tau)}$. Let τ be the bond maturity in years. The continuously compounded log-yield to maturity of the bond, $y_t^{(\tau)}$, satisfies the relation

$$y_t^{(\tau)} \equiv -\frac{1}{\tau} p_t^{(\tau)}, \tag{1}$$

where $p_t^{(\tau)}$ is the log price of the zero-coupon bond at time t - that is, $p_t^{(\tau)} = \log P_t^{(\tau)}$. It represents the per period interest rate earned from holding the bond to maturity if gains are continuously compounded. Denote the frequency (in months) at which returns are computed by h. The log forward rate at time t for loans between periods $t + \tau - h$ and $t + \tau$ is then defined as

$$f_t^{(\tau,h)} \equiv p_t^{(\tau-h)} - p_t^{(\tau)} = \tau \cdot y_t^{(\tau)} - (\tau - h) y_t^{(\tau-h)}. \tag{2}$$

The holding period return for a bond with maturity τ-years is the return of buying a bond with τ-years to maturity at time t, selling it one h-year later, at time $t + h$, as a bond with $(\tau-h)$-years to maturity, i.e.,

$$r_{t+h}^{(\tau)} = p_{t+h}^{(\tau-h)} - p_t^{(\tau)} = \tau \cdot y_t^{(\tau)} - (\tau - h)\, y_{t+h}^{(\tau-h)}, \tag{3}$$

The expected one-year holding period return on long term bonds equals the expected return on the short term bond plus the return risk premium

$$E_t \left[r_{t+h}^{(\tau)} \right] = y_t^{(h)} + \varkappa_t^{(\tau)}. \tag{4}$$

The excess return of an τ-year bond is computed as the difference between the holding period return from buying an τ-year bond at time t and selling it h-month later and the yield on a h-month bond at time t,

$$rx_{t+h}^{(\tau)} = p_{t+h}^{(\tau-h)} - p_t^{(\tau)} - h \cdot y_t^{(\tau)}, \tag{5}$$

accordingly the excess return (return risk premium) of an τ-year bond is computed as the one-year expected return in excess of the yield on a one-year bond at time t,

$$E_t\left[rx_{t+h}^{(\tau)}\right] \equiv E_t\left[r_{t+h}^{(\tau)}\right] - y_t^{(h)} = \varkappa_t^{(\tau)}. \tag{6}$$

where h is the holding period in months. When h takes one year, we obtain the usually employed one-year holding period excess bond returns.

2.2. The expectations hypothesis and risk premia

The expectations hypothesis is a convenient way to study the term structure of interest rates and also to relate macroeconomic fundamentals to the yield curve.The expectations hypothesis (EH) of the term structure of interest rates is the proposition that that the long-term interest rate is an average of expected future short-term rates (with expectations formed rationally) plus a time-independent risk premium. Fundamentally, the EH depends on the market's ability to predict the future short-term rate.

Long-term yields are determined as the average future short rate expected over the life of the bond, which are referred as the expectations hypothesis (EH) term, plus the *yield risk premium* or *term premium*,

$$y_t^{(\tau)} = \underbrace{\frac{1}{\tau}\sum_{j=0}^{\tau-1} E_t\left[y_{t+j}^{(1)}\right]}_{\text{expectations component}} + \underbrace{\varkappa_t^{(\tau)}}_{\text{yield risk premium}} \tag{7}$$

where $y_t^{(\tau)}$ is the yield at time t for a long-term bond τ-period maturity, $y_t^{(1)}$ denotes the short-term (one-year) rate, and $\varkappa_t^{(\tau)}$ denotes a constant risk premium which is allowed to vary with the maturity. Under the expectations hypothesis, the yield risk premium may be maturity-specific but does not change over time.

The relation between the return risk premium and the yield risk premium is as follows:

$$\varkappa_t^{(\tau)} = \frac{1}{\tau}\left[E_t\left(rx_{t+1}^{(\tau)}\right) + E_t\left(rx_{t+2}^{(\tau-1)}\right) + \ldots + E_t\left(rx_{t+\tau-1}^{(2)}\right)\right] \tag{8}$$

which means that the yield risk premium is the average of expected future return risk premia of declining maturity. $E_t(\cdot)$ denotes the conditional expectation given market information at time t. Notice that each of the conditional expectation terms on the right-hand side of Equation (8) are forecasts of excess bond returns, multiple steps ahead. Thus, Equation (8) shows that the excess bond return forecasts have direct implications for risk premia in yields, as well as risk premia in returns. To form an estimate of the risk-premium component in yields, $\varkappa_t^{(\tau)}$, we must form estimates of the multistep-ahead forecasts that appear on the right-hand side of Equation (8), i.e.,

$$\widehat{\varkappa}_t^{(\tau)} = \frac{1}{\tau}\left[\widehat{E}_t\left(rx_{t+1}^{(\tau)}\right) + \widehat{E}_t\left(rx_{t+2}^{(\tau-1)}\right) + \ldots + \widehat{E}_t\left(rx_{t+\tau-1}^{(2)}\right)\right] \tag{9}$$

where $\widehat{E}_t(\cdot)$ denotes an estimate of the conditional expectation $E_t(\cdot)$ formed by a linear projection. Thus, estimates of the conditional expectations are simply linear forecasts of excess returns, multiple steps ahead.

According to the expectations hypothesis of the term structure of interest rates, the yield risk premium is constant. This implies that expected excess returns are time invariant and, thus, excess bond returns should not be predictable with variables in the information set at time t. However, empirical tests of the expectations hypothesis of the term structure often rejected using a wide variety of tests and data, over a variety of time periods and monetary policy regimes, and argues that deviations from the EH reflect time-varying risk premia. The most commonly given reason for the failure of the EH is that the risk premium is not constant as the EH requires, but is time-varying. The logic underlying the theory, that expectations of future short interest rates shape the term structure of longer interest rates, is intuitive, appealing, and a common assumption in macroeconomic modeling. However, the predictability of excess returns undermines the premise that long interest rates are rational expectations of future short rates up to a constant term premium. Rather, such evidence points strongly toward time-varying risk premia.

2.3. Forecasting bond excess returns using forward rates and macro factors

Our objective is to forecast expected excess bond returns. To assess the statistical evidence on bond return predictability, we run regressions of bond excess returns at time $t+h$ on forward rates at time t. Therefore, in this paper, we consider three predictors: the forward spreads as proposed by Fama & Bliss (1987), a linear combination of forward rates as proposed by Cochrane & Piazzesi (2005), and a linear combination of macro factors, as proposed by Ludvigson & Ng (2009) and refer to them as FB, CP, and LN, respectively. The FB forward spreads are given by

$$FB_t^{(\tau,h)} = f_t^{(\tau-h,\tau)} - h \cdot y_t^{(h)}. \tag{10}$$

So, FB estimate the excess return equations

$$rx_{t+h}^{(\tau)} = \beta_0 + \beta_1 FB_t^{(\tau,h)} + \varepsilon_{t+h}^{(\tau)}, \tag{11}$$

where $\tau \in \{2, \ldots, 5\}$ denotes the vector of maturities measured in years.

In this spirit, Cochrane & Piazzesi (2005) extend the Fama & Bliss (1987) approach and run regressions of excess returns on all forward rates. CP estimate a general regression where bond excess returns are predicted by the full term structure of forward rates and the one-period yield:

$$rx_{t+h}^{(\tau)} = \beta_0^{(\tau)} + \beta_1^{(\tau)} y_t^{(1)} + \beta_2^{(\tau)} f_t^{(\tau_2)} + \ldots + \beta_5^{(\tau)} f_t^{(\tau_5)} + \varepsilon_{t+h}^{(\tau)}, \tag{12}$$

We construct the CP factor following Cochrane & Piazzesi (2005), i.e., at each time t, the average excess bond return across maturities is regressed on the one-year bond yield and the full term structure of forward rates

$$\overline{rx}_{t+1} = \gamma_0 + \gamma_1 y_t^{(1)} + \gamma_2 f_t^{(\tau_2)} + \ldots + \gamma_5 f_t^{(\tau_5)} + \upsilon_{t+h} = \boldsymbol{\gamma}' \mathbf{Z}_t + \upsilon_{t+h}, \tag{13}$$

where $\mathbf{Z}_t = \left[1, y_t^{(1)}, f_t^{(\tau_2)}, \ldots, f_t^{(\tau_5)}\right]'$ and $\overline{rx}_{t+1} = \frac{1}{4}\sum_{\tau=2}^{5} rx_{t+1}^{(\tau)}$. Then the CP factor is computed as

$$CP_t = \widehat{\gamma}_0 + \widehat{\boldsymbol{\gamma}}' \mathbf{Z}_t, \tag{14}$$

Then estimate the equation

$$rx_{t+1}^{(\tau)} = \zeta + \lambda CP_t + \epsilon_{t+1}. \tag{15}$$

CP argue that Equation (15) encompasses equation (12). Note that when the regression coefficients $\boldsymbol{\beta}' = [\beta_1, \ldots, \beta_5]' = 0$, this specification reduces to the expectation hypothesis, under which bond excess returns are unpredictable and bond risk premia are constant over time. Following Thornton & Valente (2012) and Gargano *et al.* (2017), we use this historical average of excess bond returns to serve as a natural benchmark forecasting model. Indeed, the historical average is consistent with the expectations hypothesis of the term structure of interest rates.

Ludvigson & Ng (2009) find that "real activity growth" and "inflation" factors, extracted from a large number of macroeconomic time series, have significant forecasting power for future excess returns on nominal bonds and that this predictability is above and beyond the predictive power contained in forward rates and yield spreads. Suppose we observe a $T \times M$ panel of macroeconomic variables $\{\mathbf{x}_{i,t}\}$ generated by a factor model

$$\mathbf{x}_{i,t} = \kappa_i \mathcal{F}_t + \epsilon_{i,t} \tag{16}$$

where $\mathcal{F}_t$ is an $s \times 1$ vector of common factors and $s << M$. The unobserved common factor, $\mathcal{F}_t$ is replaced by an estimate, $\widehat{\mathcal{F}}_t$, obtained using principal components analysis. The LN factor is a linear combination of the estimated principal components extracted from a dataset of 20 macroeconomic data series, $\widehat{\mathcal{F}}_t = \left[\widehat{\mathcal{F}}_{1,t}, \widehat{\mathcal{F}}_{2,t}, \widehat{\mathcal{F}}_{3,t}\right]$,

$$LN_t = \widehat{\Psi}'\widehat{\mathcal{F}}_t, \tag{17}$$

where Ψ is obtained from the projection

$$\overline{rx}_{t+1} = \psi_0 + \psi_1\widehat{\mathcal{F}}_{1,t} + \psi_2\widehat{\mathcal{F}}_{2,t} + \psi_3\widehat{\mathcal{F}}_{3,t} + \eta_{t+h}. \tag{18}$$

In the empirical analysis, we concentrate on the first 3 Principal Components $\left(\widehat{\mathcal{F}}_{1,t}, \widehat{\mathcal{F}}_{2,t}, \widehat{\mathcal{F}}_{3,t}\right)$, similar to Eriksen (2017).[2]

3. Data and empirical results

3.1. Data

Our data consist of end of the month 1- to 5-year zero-coupon yields between January, 2000 and June, 2017. This choice provides us with a panel of 210 monthly observations on 5 different yields. The data set consists of end-of- month yields of Brazilian interbank deposit future contracts (DI-futuro) collected on a monthly basis. The source of the data is the Brazilian Mercantile and Futures Exchange (BM&FBovespa), which is the entity that offers DI-futuro contracts and determines the maturities with authorized contracts. The DI-futuro contract with maturity τ is a zero-coupon future contract in which the underlying asset is the DI-futuro interest rate accrued on a daily basis, capitalized between trading period t and τ. The DI-futuro rate is the average daily rate of Brazilian interbank deposits (borowing/lending), calculated by the Clearinghouse for Custody and Settlements (CETIP) for all business days. The DI-futuro rate, which is published on a daily basis, is expressed in annually compounded terms, based on 252 business days.[3] We consider one-year holding period and construct overlapping monthly excess bond returns. This implies that h is equal to 1 in Equations (11) to (18).

Panel A in Table 1 reports descriptive statistics for the Brazilian bond excess returns based on the DI-futuro market and Fama-Bliss forward spreads along with the CP_t and LN_t factors. For each time series we report the mean, standard deviation, skewness, kurtosis and sample autocorrelation for lag-1. The summary statistics displayed in Table 1 show that excess returns are positive and highly serially correlated. As expected, the mean and standard deviation of excess returns increase

[2]The data for the LN_t factor are obtained from the Brazilian Central Bank, the FGV, the IBGE, the IPEADATA, and the Bloomberg database. The data broadly cover almost all economic categories used in Ludvigson & Ng (2009). In particular, the series include output and labor market variables, exchange rates, price indexes, income series, and the money stock. Additional details about this data set can be found in the Appendix.

[3]Additional details about this data set and the DI-futuro contract can be found in Caldeira *et al.* (2016).

with maturity, consistent with the existence of a risk premium for long maturities. Furthermore, we notice that both skewness and kurtosis decreases with respect to maturity. Both short- and long-maturity excess bond returns are very highly autocorrelated, as the first-order autocorrelations range from 0.83 (2-year) to 0.77 (5-year).

Regarding return predictors, we find that the FB factors are highly correlated with each other and strongly positively autocorrelated with first-order autocorrelation coefficients around 0.80. The CP and LN factors also exhibit high first-order autocorrelations, of 0.82 and 0.80, respectively. Panel B shows that the FB factors are strongly positively correlated with the CP factor, with correlations around 0.8, but FB factors has relatively small correlations with the LN factor. The CP factor has relatively small correlation with the LN factor, 0.51. The LN factor captures a largely orthogonal component in relation to the other predictors. As expected, excess returns are correlated with lagged CP_t factor, re-assuring that the shape of the yield curve contain information on bond risk premium. Figure 1 plots the time series of the predicted (from the predictive regressions using the CP facto) and realized holding period excess returns for 2-, 3-, 4-, and 5-year bonds. The figure shows that the CP factor is is able to predict the average excess return.

Figure 1: **Average 1-Year holding period excess return: realized and predicted**

Note: This figure plots the time series of 4 excess bond returns (in percentage) rx_{t+1} (blue continuous line) and the dashed red line in the plots refers to the predicted values from the predictive regressions using the CP factors.

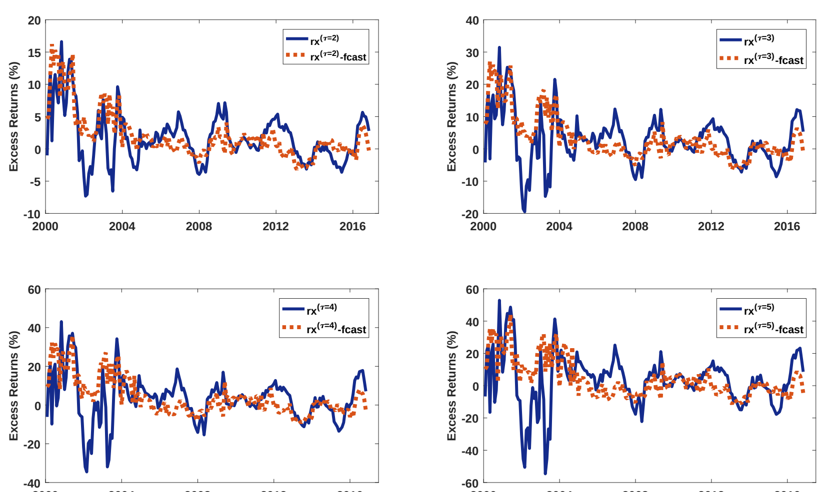

Table 1: **Summary statistics of excess returns and predictor variables**

Note: The table reports the descriptive statistics for bond excess returns computed over the different maturities, the predictor variables used in the empirical analyses (Panel A), and their contemporaneous correlations (Panel B). CP_t is the forward rate-based predictor factor from Cochrane & Piazzesi (2005) and LN_t is the macro-based factor from Ludvigson & Ng (2009). For each variable, we report means, standard deviations, skewness, and kurtosis as well as first-order autocorrelations. The sample period is 2000:01-2017:06.

	$rx_{t+1}^{(2)}$	$rx_{t+1}^{(3)}$	$rx_{t+1}^{(4)}$	$rx_{t+1}^{(5)}$	$\mathrm{FB}_t^{(2)}$	$\mathrm{FB}_t^{(3)}$	$\mathrm{FB}_t^{(4)}$	$\mathrm{FB}_t^{(5)}$	CP_t	LN_t
	Panel A: Descriptive statistics									
Mean	0.017	0.025	0.030	0.031	0.011	0.016	0.021	0.020	0.026	0.048
Std Dev	0.040	0.079	0.120	0.165	0.023	0.036	0.044	0.045	0.076	0.050
Skewness	0.784	0.387	0.039	−0.297	1.947	2.106	2.007	2.023	1.886	0.392
Kurtosis	4.815	4.602	4.303	4.115	6.482	13.15	11.29	11.51	7.399	3.315
ACF(1)	0.825	0.793	0.777	0.769	0.789	0.774	0.828	0.827	0.822	0.805
	Panel B: Correlation matrix									
$rx_{t+1}^{(2)}$	1.000									
$rx_{t+1}^{(3)}$	0.988	1.000								
$rx_{t+1}^{(4)}$	0.969	0.994	1.000							
$rx_{t+1}^{(5)}$	0.955	0.985	0.997	1.000						
$\mathrm{FB}_t^{(2)}$	0.626	0.669	0.680	0.667	1.000					
$\mathrm{FB}_t^{(3)}$	0.620	0.680	0.700	0.691	0.969	1.000				
$\mathrm{FB}_t^{(4)}$	0.629	0.686	0.725	0.723	0.902	0.949	1.000			
$\mathrm{FB}_t^{(5)}$	0.627	0.684	0.723	0.722	0.900	0.948	0.999	1.000		
CP_t	0.762	0.790	0.816	0.808	0.805	0.803	0.886	0.879	1.000	
LN_t	0.336	0.349	0.351	0.340	0.325	0.389	0.428	0.432	0.513	1.000

3.2. Statistical Evaluation

This section presents the results from our in-sample empirical analyses. We begin by considering results based on full sample estimates to remain comparable with the existing literature on expectation hypothesis and bond risk premia. The parameters in FB and CP models are estimated using 210 observations between 2000:01-2017:06 at a monthly frequency. The null hypothesis we test is no-predictability, i.e. $\beta^{(\tau_i)} = 0$, and hence regression reduces to expectation hypothesis. Tables 2 and 3 presents results from estimating predictive regressions (FB, CP, and LN) over the full range of available observations. We report slope estimates, t-statistics, and adjusted R^2 values. Given that overlapping and autocorrelated data may impact our OLS estimation, we employed a GMM (generalized method of mo- ments) estimator to correct for autocorrelation and possible heteroscedasticity (Hansen, 1982; Newey & West, 1987).

We begin with the results for the CP model computed over the full sample period presented in the left side of Table 2. The χ^2 statistic demonstrates that the EH can be rejected at the 5 per cent level for all considered maturities, indicating that bond excess returns in Brazil are somewhat predictable. We see that the CP model is able to explain 34-45% of the one-year ahead variation in bond risk premia across the maturity spectrum. Similarly to Cochrane & Piazzesi (2005) and Eriksen (2017), we obtain significant slope coefficients that are monotonically increasing with maturity. Next, we turn to our variant of the forward-spread model from Fama & Bliss (1987), in the right side of Table 2. FB is able to explain between 26% and 30% of the one-year ahead variation in bond risk premia, where the largest proportion is explained for the two- and four-year bonds. One more time the EH can be rejected at 5% level and we find evidence of time varying risk-premium across the maturity spectrum.

Table 3 reports the results from regressing one-year ahead excess-return upon CP, the forward rate-based factor from Cochrane & Piazzesi (2005), and the macro-based factor LN from Ludvigson & Ng (2009). This models are estimated in two steps. For CP model, first we estimate $\boldsymbol{\gamma}$ by running a regression of the average excess return (portfolio of all bonds) on all forward rates, and then, we estimate $\lambda^{(\tau)}$ by running four regressions of one-year ahead excess returns upon the macro factor we have attained in the first step. For LN model, first we compute the LN factor from a projection of the time-series of cross-sectional averages of the 2, 3, 4, 5 bond excess returns on three principal components obtained from a large panel of macroeconomic variables, and then, we estimate the slope coefficients by running four regressions we have attained in the first step. In both cases, the slope coefficients for the univariate models increase monotonically in the maturity of the bonds. All the coefficients are significant across all maturities and forecasting models.

For CP model, the results shows that although the coefficients differ slightly across both specifications, restricted and unrestricted models, their $\overline{R}^2$'s are almost the same for the single-factor restriction as for the unrestricted regressions. The restricted model, $\lambda^{(\tau)} \cdot \boldsymbol{\gamma}$, almost perfectly matches unrestricted coefficients. For example, a comparison of unrestricted model coefficients for a 2-year maturity $(-5.93,\ -0.81,\ 2.38,\ -1.50,\ 3.13,\ -2.77)$ with coefficient implied from the restricted model $(-13.97,\ -2.05,\ 5.16,\ -4.40,\ 10.15,\ -7.96) \times 0.38 = (-5.31,\ -0.78,\ 1.96,\ -1.67,\ 3.80,\ -3.02)$. Both models thus have similar explanatory power, allowing us to focus on the more parsimonious restricted specification.

Now, we turn to the macro-factor from Ludvigson & Ng (2009). In general, our results suggest that macro factors do contribute substantially to the understanding of the dynamics of excess-return in Brazilian fixed income market. Specifically, LN is able to explain between 30% and 47% of the one-year ahead variation in excess-returns, where the largest proportion is explained for the four- and five-year bonds. As a last step, we consider a two-factor specification using CP and LN that investigates whether macro-factors contain information about excess returns that is distinguishably different from that contained in the yield curve. As it turns out, both are individually strongly significant for all bond maturities, suggesting that they capture quite distinct aspects of the set of risks that governs the time-variations in excess returns. Similar to Ludvigson & Ng (2009) and Eriksen (2017), we find CP and LN to contain complementary information as both factors remain significant and jointly produce adjusted R^2 values larger than their individual values for all maturities and both factors remains significant at 5% level for all maturities.

Table 2: **Estimates of Cochrane and Piazzesi and Fama-Bliss predictive regressions from 2000:01 to 2017:06.**

Note: The table reports the estimates of Cochrane & Piazzesi (2005) predictive regression (unrestricted model). The regression equation for unrestricted model is

$$rx_{t+1}^{(\tau)} = \beta_0^{(\tau)} + \beta_1^{(\tau)} y_t^{(1)} + \beta_2^{(\tau)} f_t^{(2)} + \beta_3^{(\tau)} f_t^{(3)} + \beta_4^{(\tau)} f_t^{(4)} + \varepsilon_{t+1}^{(\tau)}.$$

And, the Fama-Bliss predictive regression model. The regression equation for FB model is

$$rx_{t+1}^{(\tau)} = \beta_0 + \beta_1 fs_t^{(\tau)} + \varepsilon_{t+1}^{(\tau)}.$$

Point estimates are reported with Newey & West (1987) standard errors, accounting for conditional heteroscedasticity and serial correlation up to twelve lags, in parentheses. . *,**,*** indicate that the slope coefficients are statistically significant at 10% , 5%, and 1% level, respectively. $\chi^2(5)$ is the Wald statistic that tests whether the slope coefficients are jointly zero (the 5% and 1% critical values are 11.1 and 15.1, respectively). The parameters are estimated using 210 observations between 2000:01 and 2017:06. $\overline{R}^2$ refers to adjusted R^2.

Maturidade (τ-years)	Cochrane & Piazzesi - Unrestricted Model								Fama-Bliss Model		
	β_0	β_1	β_2	β_3	β_4	β_5	$\overline{R}^2$	$\chi^2(5)$	β_0	β_1	$\overline{R}^2$
$\tau = \mathbf{2}$	-5.93^{***}	-0.806^{***}	2.384^{***}	-1.495^{***}	3.131^{***}	-2.773^{***}	0.452	103.61	0.193	1.218^{***}	0.293
	(2.139)	(0.167)	(0.357)	(0.619)	(1.617)	(1.472)			(0.826)	(0.433)	
$\tau = \mathbf{3}$	-11.886^{***}	-1.593^{***}	4.099^{***}	-2.722^{***}	7.382^{***}	-6.347^{***}	0.408	93.37	-0.199	1.644^{***}	0.267
	(4.712)	(0.409)	(0.689)	(1.586)	(3.716)	(3.295)			(1.78)	(0.764)	
$\tau = \mathbf{4}$	-16.988^{***}	-2.509^{***}	6.123^{***}	-5.34^{***}	12.788^{***}	-9.962^{***}	0.385	81.69	-1.583	2.319^{***}	0.304
	(7.527)	(0.787)	(1.105)	(2.93)	(5.988)	(5.211)			(3.276)	(0.967)	
$\tau = \mathbf{5}$	-21.024^{***}	-3.275^{***}	8.032^{***}	-8.027^{***}	17.311^{***}	-12.737^{***}	0.343	59.92	-2.476	2.979^{***}	0.267
	(10.472)	(1.181)	(1.685)	(4.321)	(8.193)	(7.019)			(4.478)	(1.366)	

Table 3: **In-sample estimates with CP and LN factors**

Note: This table reports estimates of the slope coefficients from regressing one-year ahead excess-return upon CP_t, the forward rate-based factor from Cochrane & Piazzesi (2005), and the macro-based factor LN_t from Ludvigson & Ng (2009). Panel A the presents estimates of the CP_t predictor computed from a projection of the time series of cross-sectional averages of the 2, 3, 4, 5 excess returns on the 1, 2, 3, 4 and 5 year forward rates. Panel B presents the univariate predictive regression results for monthly excess returns upon CP_t or LN_t factors. The LN_t predictor is computed from a projection of the time-series of cross-sectional averages of the 2, 3, 4, 5 bond excess returns on five principal components obtained from a large panel of macroeconomic variables. Newey & West (1987) standard errors, accounting for conditional heteroscedasticity and serial correlation up to twelve lags, are presented in parentheses. *,**,*** indicate that the slope coefficients are statistically significant at 10%, 5%, and 1% level, respectively. $\overline{R}^2$ denotes the full sample adjusted coefficient of determination. The parameters are estimated using 210 observations between 2000:01 and 2017:06.

	Panel A: Cochrane & Piazzezi - regression for $\overline{rx}_{t+1}$							
	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5	$\overline{R}^2$	$\chi^2(5)$
OLS Estimates	-13.957^{***}	-2.045^{***}	5.159^{***}	-4.396^{*}	10.153^{*}	-7.955^{***}	0.394	82.068
	(5.596)	(0.746)	(1.409)	(2.073)	(4.453)	(3.931)		

	Panel B: Preditive Regressions						
Maturities (τ-years)	CP_t	SE	$\overline{R}^2$	LN_t	SE	$\overline{R}^2$	$\overline{R}^2$ (CP_t + LN_t)
$\tau = \mathbf{2}$	0.378^{***}	(0.066)	0.438	0.327^{***}	(0.153)	0.309	0.596
$\tau = \mathbf{3}$	0.785^{***}	(0.189)	0.415	0.735^{***}	(0.312)	0.379	0.653
$\tau = \mathbf{4}$	1.241^{***}	(0.360)	0.401	1.235^{***}	(0.485)	0.438	0.707
$\tau = \mathbf{5}$	1.596^{***}	(0.545)	0.356	1.703^{***}	(0.634)	0.469	0.712

In summary, our in-sample estimation results indicates that we did not find evidence supportive of the expectations hypothesis theory for Brazilian yield curve. An important implication of this is that Brazilian central banks have a low ability to influence long rates through monetary policy adjustments of short rates. This is, for example, of particular relevance to those investment decisions based on interest rates at the longer end of the maturity spectrum. The Brazilian interest rates fail to support the expectations hypothesis possibly due to the times of high volatility, resulting in large deviations between the expected and the actual spread. This is consistent with previous studies such as Beechey *et al.* (2009), which found that the EH did not hold in developing countries due to high volatility interest rates.

3.3. Out-of-sample forecasting

In this section, we evaluate the ability of the bond return prediction models from Section 2.3 to accurately forecast bond risk premia in an out-of-sample setting using information available at the time of the forecast only. As argued by Thornton & Valente (2012), Sarno *et al.* (2016), and Eriksen (2017), among others, a good in-sample fit does not necessarily translate into positive out-of-sample performance.

3.3.1. Statistical evaluation

To access the pseudo real-time performance of the bond return prediction models, we consider a statistical evaluation of the predictive accuracy of the out-of-sample forecasts relative to a recursively updated expectations hypothesis (EH) benchmark computed as a recursively updated projection of bond excess returns upon a constant. For that, we use 2000:01-2010:12 as our initial warm-up estimation sample and 2011:01-2017:06 as the forecast evaluation period. The forecasts are generated recursively using an expanding window of observations, where model parameters and predictor variables are updated recursively prior to each forecast as well. Importantly, we rely on historically available information only, information available at time t to compute return forecasts for period $t+1$, to mimic a real-time forecasting environment and avoid concerns of look-ahead bias induced by full sample parameters.

We follow Eriksen (2017) and consider two measures of statistical significance well-known to the literature. First, to measure the relative performance of the FB, CP, and LN models with respect to the expectations hypothesis, we use the relative mean square forecast error (rMSFE). The MSFE is computed as

$$\text{MSFE}_m^{(\tau)} = \frac{1}{Ts}\sum_{t=1}^{Ts}\left(rx_{t+1}^{(\tau)} - \widehat{rx}_{t+1,m}^{(\tau)}\right)^2$$

where $rx_{t+1}^{(\tau)}$ and $\widehat{rx}_{t+1,m}^{(\tau)}$ denote the forecast from the ith candidate model and the EH benchmark model, respectively, and Ts is the number of out-of-sample forecasts. Next, following Campbell & Thompson (2008), we provide for each bond maturity and model, an out-of-sample R^2 relative to the EH benchmark model given as

$$R^2_{\text{OoS},m} = 1 - \frac{\sum_{t=1}^{Ts}\left(rx_{t+1}^{(\tau)} - \widehat{rx}_{t+1,m}^{(\tau)}\right)^2}{\sum_{t=1}^{Ts}\left(rx_{t+1}^{(\tau)} - \widehat{rx}_{t+1,\text{EH}}^{(\tau)}\right)^2}$$

whereby a positive $R^2_{\text{OoS},m}$ indicates that the point forecasts associated with the model m are, on average, more accurate than the EH benchmark forecasts. To gauge the significance of R_{OoS}, we use the test for equal predictive accuracy suggested by Clark & West (2007).

Table 4 presents results from the statistical evaluation of the models against the EH benchmark across the four bond maturities. We see that FB and CP factor performs poorly against the EH benchmark for the full spectrum of maturities, where it realizes negative R_{OoS}, -2.80% to -1.40%.

We find little evidence that individual models considered are able to improve on the predictive accuracy of the EH model, although the LN fare better for the longer bond maturities. Conversely, considering a two-factor model including CP and LN results in consistently positive R_{OoS} in the range of 1.42% to 4.90% over the spectrum of bond maturities, signaling a forecasting performance superior to the simple EH benchmark of constant expected returns, which are all significantly positive at 10% confidence at least according to the Clark & West (2007) tests. Consequently, CP and LN appear to contain complementary information that results in significant out-of-sample forecasting gains.

Table 4: **Out-of-sample predictive performance for excess returns from 2011:01 to 2017:04**

Note: This table reports the out-of-sample results from forecasting one-year ahead excess return using FB and CP models relative to the expectations hypothesis (EH) benchmark. First, the table reports the relative MSFE of the considered models over the MSFE of the EH. Next, shows the R^2_{OoS} is the out-of-sample R^2 suggested in Campbell & Thompson (2008). Bold entries indicates statistic significance at 10% level based on Clark & West (2007) test of equal predictive ability. The sample starts on January 2000 and the evaluation period is January 2011 to June 2017.

Maturidade (τ-years)	FB_t forward spread		CP_t factor		LN_t factor		$(CP_t + LN_t)$ factors	
	rMSFE	R^2_{OoS} (in %)	rMSFE	R^2_{OoS} (in %)	rMSFE	R^2_{OoS} (in %)	rMSFE	R^2_{OoS} (in %)
$\tau = \mathbf{2}$	1.071	−1.480	1.068	−1.400	1.007	−1.417	0.994	1.243
$\tau = \mathbf{3}$	1.079	−1.650	1.062	−1.270	1.001	−0.101	0.985	**3.020**
$\tau = \mathbf{4}$	1.099	−2.080	1.074	−1.530	0.994	1.292	0.979	**4.126**
$\tau = \mathbf{5}$	1.127	−2.690	1.092	−1.920	0.990	1.936	0.976	**4.789**

In summary, this study find evidence not supportive to expectations hypothesis for Brazil, for the 2000 - 2016 period. The rejection of the simple expectations theory is consistent with the hypothesis of time-varying term premia. These results differ from other research focusing on Brazilian market (Lima & Issler, 2003, 2007; Tabak, 2009). The Expectations Hypothesis has received a great deal of attention in the empirical literature; however, the findings are not uniform, depending often on the precise implication tested, the maturities of the yield curve examined or the period under study.

4. Conclusion

The expectation hypothesis (EH) plays important roles in economics and finance and, not surprisingly, has been widely tested using a variety of tests and datam mainly for developed markets. This study analyze the expectation hypothesis and investigates the predictive power of term structure of interest rates and macroeconomic factors for excess bond returns in the Brazilian fixed income market. As predictors we use the forward spread variable of Fama & Bliss (1987), the Cochrane & Piazzesi (2005) combination of forward rates, and the Ludvigson & Ng (2009) macro factors. The results show that the no-predictability benchmark is difficult to beat in by either of the competing forward-rate models and macroeconomic factors.

Our empirical findings indeed suggest that the Brazilian interest rates fail to support the expectations hypothesis. We find that excess returns are indeed predictable, although the predictability is not as high as documented in previous literature (for example, Gargano *et al.* , 2017; Eriksen, 2017, and references therein). We show that macroeconomic factors have an important role in forecasting excess bond returns in Brazilian fixed income market. Macro risks are unspanned in yields but help predict bond returns, consistent with the evidences in recent literature. Importantly, we also find that a two factor model including CP and LN factors significantly improve the predictive power for excess returns. The forecasts turn out that the expectations hypothesis fails in the Brazilian fixed income market.

Future studies can extend our empirical application in several directions. First, the applied framework can be generalized to allows for time varying regression parameters and stochastic volatility dynamics. Second, we can investigates the economic gains to an investor who exploits the predictability of bond excess returns relative to the no-predictability alternative consistent with the expectations hypothesis. Third, can be interesting examine the usefulness of other unspanned return-forecasting factors, for example, international factors. We leave this open question for future research.

Appendix: Data appendix

This Appendix lists the 20 macroeconomic time series used to construct the latent common factors underlying the recursive construction of the real-time variant of the LN_t factor. All series are obtained from the Brazilian Central Bank, the FGV, the IBGE, the IPEADATA, and the Bloomberg database and covers the months 2000:M1 to 2017:M6. For each variable, Table 5 reports variable description and the transformation code (Tcode) used to ensure covariance stationarity of the underlying data series (following Eriksen, 2017). In particular, if $x_{i,t}$ is the original untransformed series, the transformation codes are: 1 - no transformation (levels), $z_{i,t} = x_{i,t}$; 2 - first difference, $z_{i,t} = x_{i,t} - x_{i,t-1}$; 3 - second difference, $z_{i,t} = x_{i,t} - x_{i,t-2}$; 4 - logarithm, $z_{i,t} = \log x_{i,t}$; 5 - first difference of logarithm, $z_{i,t} = \ln x_{i,t} - \ln x_{i,t-1}$; 6 - second difference of logarithm $z_{i,t} = \ln x_{i,t} - \ln x_{i,t-2}$

Table 5: **Series used in the construction of the LN_t factor**

Note: The table reports variable descriptions and transformation codes indicating the transformation applied to the series. The data covers the months from 2000:M1 to 2017:M6.

Serie ID	Tcode	Description	Source
IPCA	6	Consumer Price Index	IPEADATA
SELIC	2	Effective interest rates	BACEN
PMCP	1	NAPM commodity prices index	FRED
CBDR	2	Central bank discount rate	BACEN
IBOV	5	Ibovespa Index	Bloomberg
M2SL	6	Brazil Money Supply: M2 Brazil	BACEN
PAINC	6	Average Income	FGV
INDPRO	5	Industrial Production Index (general)	FGV
UNRATE	2	IBGE Brazil Unemployment Rate	IBGE
HOUST	4	Housing starts	FGV
IPA	6	Producer Price Index	IBGE
PINCOME	6	Real Personal income	FGV
M1SL	6	Brazil Money Supply: M1 Brazil	BACEN
OILPRICE	5	Spot oil price: west Texas intermediate	FRED
GS5	2	5-year treasury constant maturity rate	Bloomberg
EXBRLUS	5	USD-BRL Foreign Exchange Rate	IPEADATA
CAGED	2	Brazil Unemployment Statistic Total	FGV
IBC-BR	5	Index of Economic Activity of the Central Bank	BACEN
UCI	1	Capacity Utilization	IPEADATA
CDI1	2	1 Day DI Futures Contracts	BACEN

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