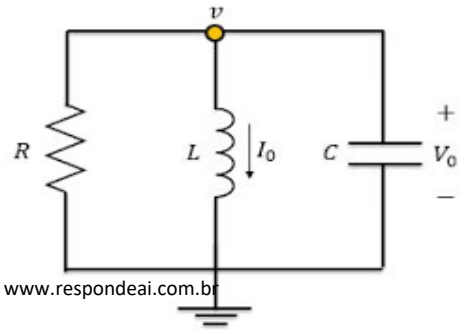
CIRCUITOS RLC

DETERMINAÇÃO DAS CONDIÇÕES INICIAIS

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➢ Resposta Natural ou Transitória – Circuitos de 2ª. Ordem - RLC

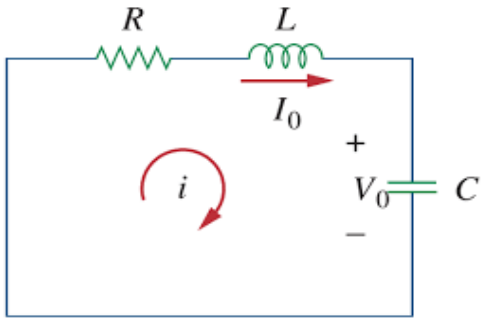


Fonte: www.respondeai.com.br

$$v_C(t_o) = V_o$$

e

$$i_L(t_o) = I_o$$



Fonte: paginapessoal.utfpr.com.br

$$\frac{v}{R} + \frac{1}{L}\int v dt + Io + C\frac{dv}{dt} = 0$$

$$R.i + \frac{1}{C}\int i dt + Vo + L\frac{di}{dt} = 0$$

Aplicando $\frac{d}{dt}$ para eliminar a Integral, e ajustando, se tem:

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0$$

Para obter a solução se vai assumir que:

$$v(t) = A.e^{-st}$$

$$\mathrm{i}(t) = A.e^{-st}$$

Substituindo nas equações diferenciais, se obtém:

$$A.e^{-st}\left(s^2 + \frac{1}{RC}.s + \frac{1}{LC}\right) = 0$$

$$A.e^{-st}\left(s^2 + \frac{R}{L}.s + \frac{1}{LC}\right) = 0$$

Equação Característica

Para resolver a equação, lança-se mão da equação característica, a qual tem duas raízes S_1 e S_2, tais como:

$$S_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$S_2 = -\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Sendo:

$$\alpha = \frac{R}{2L}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$S_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$S_2 = -\frac{1}{2RC} - \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

Sendo:

$$\alpha = \frac{1}{2RC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

De forma genérica, se tem: $S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

α => Frequência Neperiana ou Fator de Amortecimento (np/s)

ω0 => frequência Natural ou Frequência Natural Não-Amortecida

Para α > ω0

$\Rightarrow S_1 \neq S_2 \in \mathcal{R}$

$v(t) = A_1 e^{-s_1 t} + A_2 e^{-s_2 t}$ $\quad i(t) = B_1 e^{-s_1 t} + B_2 e^{-s_2 t}$ **Superamortecido**

Para α = ω0

$\Rightarrow S_1 = S_2 \in \mathcal{R}$

$v(t) = e^{-\alpha t}(A_1 t + A_2)$ $\quad i(t) = e^{-\alpha t}(B_1 t + B_2)$ **Criticamente Amortecido**

Para $\Rightarrow \alpha < \omega_0$

$\Rightarrow S_1 = \overline{S_2} \in \mathbb{C}$

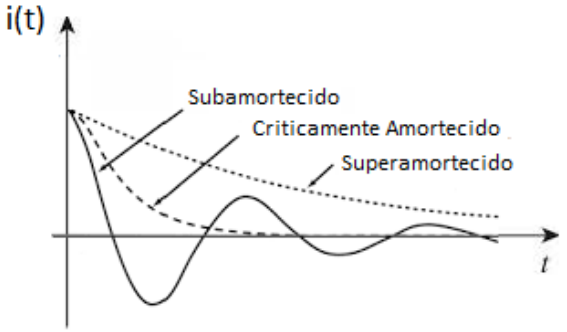
$v(t) = e^{-\alpha t}(A_1 \sin \omega_d t + A_2 \cos w_d t)$ $\quad i(t) = e^{-\alpha t}(B_1 \sin \omega_d t + B_2 \cos w_d t)$

Subamortecido

$S_{1,2} = -\alpha \pm j\omega_d$ $\quad sendo\ \ j = \sqrt{-1}$

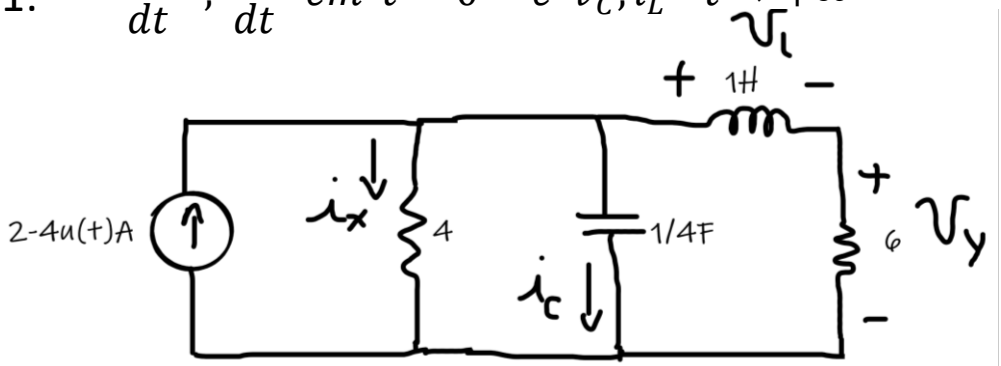
$\Rightarrow \omega_d = \sqrt{\omega_0^2 - \alpha^2}$

Graficamente, se tem:

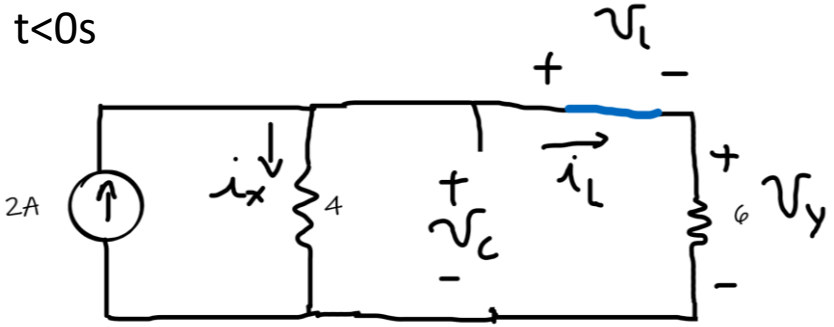


Em virtude das duas constantes nas equações, se tem a necessidade de conhecer: $\left.\frac{dv}{dt}\right|_{t_o^+}$ ou $\left.\frac{di}{dt}\right|_{t_o^+}$

- Exemplo1: $\frac{di_x}{dt}, \frac{dv_L}{dt}$ em $t = 0^+$ e v_C, i_L $t \to +\infty$



t<0s

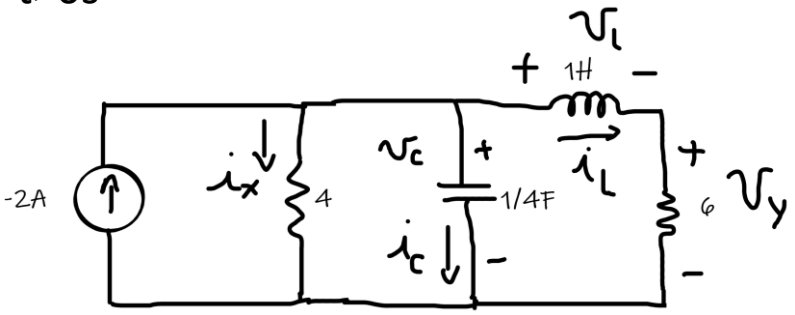


$t = 0^-$

$$i_L(0^-) = \frac{4}{4+6} x2 = 0{,}8A = 800mA$$

$$v_C(0^-) = 6.\, i_L(0^-) = 4{,}8V$$

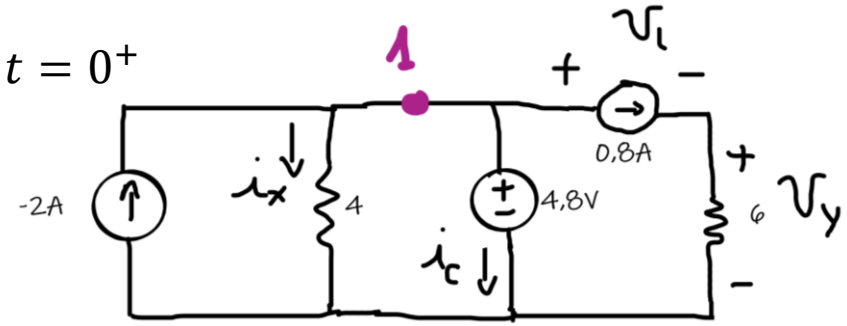
t>0s



$$i_L(0^-) = i_L(0) = i_L(0^+) = 0{,}8A$$

$$v_C(0^-) = v_C(0) = v_C(0^+) = 4{,}8V$$

$t = 0^+$



$i_L(0^+) = 0{,}8A$ $\qquad$ $\frac{di_x}{dt}, \frac{dv_L}{dt}$ em $t = 0^+$??

$v_C(0^+) = 4{,}8V$ $\qquad$ $C = {}^1/_4 F$ $\quad e \quad$ $L = 1H$

Nó 1 $\Rightarrow$ $i_x + i_C + i_L = -2 \Rightarrow$ $\quad i_C = -2 - i_x - i_L \Rightarrow$ $\quad i_X = \frac{v_C}{4} \Rightarrow$ $\quad i_X(0^+) = \frac{v_C(0^+)}{4} \Rightarrow$

$\Rightarrow i_X\,(0^+) = \frac{4{,}8}{4} = 1{,}2A \Rightarrow$ $\quad i_C(0^+) = -2 - 1{,}2 - 0{,}8 = -4A$

$$i_X = \frac{v_C}{4} \Rightarrow \frac{di_x}{dt} = \frac{1}{4} x \frac{dv_C}{dt} = \frac{1}{4} x \frac{i_C}{C} \Rightarrow \frac{di_x}{dt}(0^+) = \frac{1}{4} x \frac{i_C\,(0^+)}{C} \Rightarrow \frac{di_x}{dt}(0^+) = \frac{1}{4} x \frac{-4}{{}^1/_4} \Rightarrow \boxed{\frac{di_x}{dt}(0^+) = -4A/s}$$

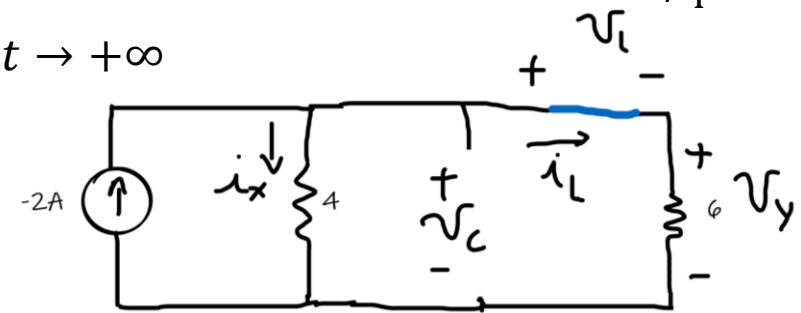
Na Malha do Indutor, se tem: $v_L + v_Y - v_C = 0 \Rightarrow v_L = v_C - v_Y \Rightarrow$

Ainda, $v_Y = 6.i_L \Rightarrow$ $\quad v_L = v_C - 6.i_L \Rightarrow$ $\quad v_L(0^+) = v_C(0^+) - 6.i_L(0^+) \Rightarrow$ $\quad v_L(0^+) = 4{,}8 - 6.0{,}8 = 0V$

Sendo: $v_L = v_C - 6.i_L \Rightarrow$ $\quad \frac{dv_L}{dt} = \frac{dv_C}{dt} - 6.\frac{di_L}{dt} \Rightarrow$ $\quad \frac{dv_L}{dt}(0^+) = \frac{i_C(0^+)}{C} - 6.\frac{v_L(0^+)}{L} \Rightarrow$

$$\frac{dv_L}{dt}(0^+) = \frac{-4}{{}^1/_4} - 6.\frac{0}{1} \Rightarrow \quad \boxed{\frac{dv_L}{dt}(0^+) = -16V/s}$$

$t \to +\infty$



$$i_L(+\infty) = \frac{4}{4+6} x - 2 = -0{,}8A = -800mA$$

$$v_C(+\infty) = 6.i_L(+\infty\ ) = -4{,}8V$$